Ivars Peterson's **MathLand**

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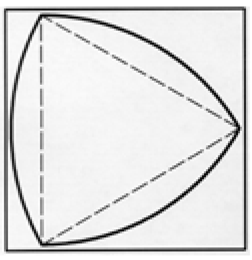
Rolling with Reuleaux

Why is the cover of a manhole round? The usual answer is that a circular lid, unlike a square or hexagonal cover, won't fall through the opening. There's no way of orienting a round lid so that it fits through a hole of the same geometry but slightly smaller size.

The circle works because it has a constant width. This width is defined as the distance between a pair of parallel lines touching the curve on opposite sides. For a circle, the width is simply the circle's diameter. That's why wheels and cylindrical rollers produce a smooth ride on a flat surface.

Unlike a circle, an ellipse doesn't have the same width in all directions. So an elliptical lid could easily fall through an elliptical hole, and an object riding on elliptical rather than cylindrical rollers would jiggle up and down.

However, the circle isn't the only curve of constant width. There is actually an infinite number of such curves, any one of which could form a manhole lid or the cross section of a roller that gives as smooth a ride as a cylinder.



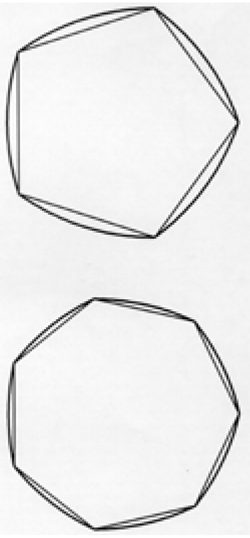
Reuleaux triangle.

The simplest such curve is known as the Reuleaux triangle, named after engineer Franz Reuleaux, who taught in Berlin during the late nineteenth century. One simple way to generate this figure is to start with an equilateral triangle, then draw three arcs of circles, with each arc having as its center one of the triangle's corners and as its endpoints the other two corners.

The resulting "curved triangle," as Reuleaux termed it, has a constant width equal to the length of the interior triangle's side. This shape, with rounded corners, may be familiar as the cross section of a bottle of NyQuil or Pepto-Bismol. Its most prominent and successful application may well be in the Wankel rotary internal combustion engine, which powers several types of cars manufactured by Mazda. The engine features a curved, triangular rotor turning in a specially shaped housing.

Like a circle, a Reuleaux triangle fits snugly inside a square having sides equal to the curve's width no matter which way the triangle is turned. Indeed, the rounded triangle can rotate freely inside the square without ever having any room to spare. .

Interestingly, as it rotates, the curved figure traces a path that eventually covers just about every part of the square (except for a little rounding at the corners). This property is the basis for an ingenious rotary drill that, constrained by a special guide plate, bores square holes.



Reuleaux curves based on the pentagon and heptagon.

It's possible to construct a curve of constant width not only from an equilateral triangle but also from any polygon with an odd number of sides. Thus, one can readily obtain a curved pentagon, heptagon, and so on. Some coins have a rounded heptagonal shape that allows their use in slot machines designed for ordinary coins. Drills shaped like curved heptagons produce hexagonal holes.

The Reuleaux curves described so far have corners -- points where two sides meet at an angle. However, curves of constant width having rounded corners can be readily constructed from the angular forms. Moreover, a curve of constant width need not be symmetrical or even consist of circular arcs. So there's an unlimited number of curves of constant width, and the Reuleaux triangle happens to be the family member of least area.

Why can't Reuleaux polygons be used in place of wheels? The trouble is that these polygons don't have a fixed center of rotation. The hub of a circular wheel, for example, stays a fixed height above the ground, allowing smooth, horizontal motion. In contrast, the center of, say, a Reuleaux triangle wobbles as the curve rotates. That doesn't matter for rollers laid down on a surface to ease the passage of a heavy load, but it does matter if the roller or wheel has a fixed axis. That's also why the drill for cutting square holes requires a special "floating" chuck to hold the drill.

References:

Gardner, Martin. 1969. *The Unexpected Hanging and Other Mathematical Diversions.* New York: Simon and Schuster.

______. 1963. Mathematical games: Curves of constant width, one of which makes it possible to drill square holes. *Scientific American* 208(February):148-156.

Rademacher, Hans, and Otto Toeplitz. 1990. *The Enjoyment of Mathematics.* New York: Dover.

Smith, Scott. 1993. Drilling square holes. *Mathematics Teacher* 86(October):579-583.

Illustrations of Reuleaux curves generated using Mathematica 3.0.

A web site devoted to the history and design of the Reuleaux roller is available at http://commando.me.berkeley.edu/~willchui/main.html. After learning about its unique shape, you can design your own Reuleaux disk and, if you're lucky, have one automatically manufactured to your specifications.

Additional information can be found at the Math Forum site: http://forum.swarthmore.edu/.

Comments are welcome. Please send messages to Ivars Peterson at ip@scisvc.org.

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Ivars Peterson is the mathematics and physics writer at *Science News* (http://www.sciencenews.org/). He is the author of *The Mathematical Tourist, Islands of Truth, Newton's Clock,* and *Fatal Defect.* His current work in progress is *Adventures in Mathland: The Jungles of Randomness* (to be published in 1997 by Wiley).